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Thermodynamic properties of a two-dimensional electron gas in the presence of a weak spatially modulated periodic potential

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Abstract. A systematic investigation on the influence of an additional periodic modulation potential which is weak, either electric or magnetic in nature, and spatially modulated along one dimension, on the equilibrium thermodynamic properties of a two-dimensional electron gas in an externally applied magnetic field is presented. The application of such an additional modulation potential results in a broadening of the Landau level energy spectrum into bands whose widths oscillate as a function of the externally applied magnetic field. Such oscillations are found to reflect the commensurability of the two different length scales present in the system, namely the cyclotron diameter at the Fermi level and the period of the modulation. We show that such commensurability effects are also to be found in all thermodynamic quantities of the system. They appear at low magnetic fields as an amplitude modulation of the well-known de Haas–van Alphen-type oscillations, familiar from the homogeneous two-dimensional electron gas system in an external magnetic field, which may or may not be resolved depending on temperature and are only weakly dependent on temperature. Their origin lies in the oscillations occurring in the bandwidths and they are consequently completely different in origin from the usual de Haas–van Alphen-type oscillations. In particular, we show that commensurability oscillations are to be found in the chemical potential, Helmholtz free energy, internal energy, electronic entropy, electronic specific heat, orbital magnetization and orbital magnetic susceptibility of such weakly modulated systems. We find that the resulting commensurability oscillations in each thermodynamic function exhibit well-defined phase relations between the electric and magnetic modulations except in the case of the orbital magnetization and the orbital magnetic susceptibility. Explicit asymptotic expressions for the chemical potential, Helmholtz free energy and orbital magnetization, in the quasi-classical limit of small magnetic fields and small but finite temperatures, are also given.

1. Introduction

For the best part of a decade now, the influence of an additional one-dimensional periodic spatial modulation potential, which is weak, on a two-dimensional electron gas (2DEG) system in a uniform quantizing magnetic field (hereafter referred to as the two-dimensional Landau system (2DLS)) has been extensively investigated. Initially such work focused on a modulation potential which was *electric* in nature [1–8] while more recently such attention shifted to a modulation potential which was either solely *magnetic* in nature [9–15] or a combination of the two types [16–18].

Modulated systems have been able to command such attention since the application of an additionally applied modulation potential to the 2DLS lifts the degeneracy of the Landau level (LL) energy spectrum and broadens these levels into bands, in a non-uniform way, whose widths oscillate with the externally applied magnetic field [8, 9] and therefore allow for interesting commensurability effects. This non-uniform broadening of the levels, being the pertinent feature of such modulated systems, initially found currency in the discovery of new magnetic-field-dependent oscillations, at low fields, in the magnetoresistance due to an electrostatic modulation potential [1, 7] and subsequently more recently due to a magnetic modulation potential [13–15]. These additional oscillations reflect the commensurability of two different length scales present in such a system, namely the cyclotron diameter at the Fermi level

$$2R_c = 2\sqrt{2\pi n_s}\ell^2$$

(here n_s is the areal density of electrons while $\ell = \sqrt{\hbar/(eB)}$ is the magnetic length where B is the strength of the uniform magnetic field and e is the electronic charge) which represents the natural length scale of the two-dimensional Landau system, and the period of the modulation a which is an additional length scale introduced into the system by the modulation. Consequently, such oscillations are completely different in origin from those of the usual de Haas–van Alphen-type (dHvA-type) oscillations, which occur at higher magnetic fields and result from the formation of discrete energy levels due to the quantizing magnetic field, and are well known [19, 20].

In this paper we show systematically that these new oscillations at low magnetic fields, now known as 'Weiss oscillations' when seen in the magnetoresistance and Weiss-type oscillations when seen in other properties of the system, also manifest themselves in all thermodynamic quantities of the system. In particular, we show that Weiss-type oscillations are to be found in the chemical potential, Helmholtz free energy, internal energy, electronic entropy and electronic specific heat, and are particularly evident in the orbital magnetization and the orbital magnetic susceptibility of such weakly modulated systems. Clear comparisons between the two different types of modulation potential present (i.e. either *electric* or *magnetic*) are given and contact between the modulated 2DLSs and that for the corresponding unmodulated system is made. Interestingly, we find that for the Weiss-type oscillations obtained for electric and magnetic modulations, definite phase relations exist between some of the thermodynamic functions while apparently not between others. Furthermore, in the limit for small magnetic fields and low temperatures, asymptotic expressions for the chemical potential, Helmholtz free energy and orbital magnetization are given that show explicitly both the Weiss- and dHvA-type oscillations.

The work here not only extends upon a brief account of some of the equilibrium thermodynamic properties (namely the chemical potential, orbital magnetization, orbital magnetic susceptibility and electronic specific heat) touched on by Peeters and Vasilopoulos [21] for a 2DLS subjected to an additional *electric* modulation potential, but, for the first time presents the equilibrium thermodynamic properties of a 2DLS subjected to an additional *magnetic* modulation potential which is weak. In the absence of a modulation potential, the thermodynamic properties of the 2DLS have been calculated most completely, in a series of papers in the mid-1980s, by Zawadzki. Initially he used a density of states (DOS) obtained from the simplest model for a 2DEG in a perpendicular magnetic field with no broadening, namely a DOS in the form of Dirac delta functions, to calculate the chemical potential and orbital magnetization [22] and the magnetization and magnetic susceptibility (both orbital and spin parts) asymptotically in the limit of very small magnetic fields [23]. Later on, the work was extended to a DOS which contained broadening in the form of a sum of Gaussian peaks in order to calculate the chemical potential, electronic entropy, electronic specific heat and orbital magnetization [24, 25].

2. The energy spectrum

We consider a 2DEG which is confined to the (x, y) plane and which is subjected to a uniform quantizing magnetic field $\boldsymbol{B} = B\hat{e}_z$. In the Landau gauge the unperturbed single-electron Hamiltonian for the 2DLS may be written as

$$H_0 = -\frac{\hbar^2}{2m_b}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}m_b\omega_c^2(x - x_0)^2. \tag{2.1}$$

Here m_b is the effective band mass of the electron, $\omega_c = eB/m_b$ is the cyclotron frequency while $x_0 = k_y\ell^2$ is the centre coordinate with k_y corresponding to the wavenumber along the y-direction. The normalized single-electron eigenfunctions corresponding to (2.1) are given by $\psi_{n,k_y}(x, y) = \exp(\mathrm{i}k_y y)\phi_n(x - x_0)/\sqrt{L_y}$. Here the $\phi_n(x - x_0)$ are the well-known linear harmonic oscillator eigenfunctions centred at x_0, n corresponds to the LL index while L_y is the normalization length in the y-direction. The associated eigenvalues for the 2DLS are given by $\varepsilon_{n,k_y} \equiv \varepsilon_n = (n + 1/2)\hbar\omega_c$, which are degenerate in the quantum number k_y, and form equally spaced LLs separated by an amount given by $\hbar\omega_c$.

For the case of a spatial electric modulation potential, $V(x)$, in the x-direction, the single-electron Hamiltonian is given by $H^{\{\mathrm{e}\}} = H_0 + V(x)$. Here the letter 'e' is used to denote the electric case. It will be modelled by a simple sinusoidal potential of the form $V(x) = V_\mathrm{e}\cos(Kx)$ such that the modulation potential is weak, i.e. $V_\mathrm{e} \ll \varepsilon_F^0$. Here V_e is the constant amplitude of the electric modulation potential, $\varepsilon_F^0 = \hbar^2k_F^2/(2m_b)$ is the Fermi energy at zero magnetic field and temperature while $k_F = \sqrt{2\pi n_s}$ is the magnitude of the Fermi wave vector in two dimensions. Lastly $K = 2\pi/a$. Choosing the Landau gauge, the eigenvalues for this weakly perturbed system can therefore be found using first-order perturbation theory. The result is [26]

$$E_{n,k_y}^{\{\mathrm{e}\}} = (n + 1/2)\hbar\omega_c + U_n\cos(Kx_0). \tag{2.2}$$

Here $U_n = V_\mathrm{e}\exp(-\mathcal{X}/2)L_n(\mathcal{X})$, $\mathcal{X} = (K\ell)^2/2$ while $L_n(x)$ is a Laguerre polynomial.

The pertinent feature of the addition of the modulation potential is that it lifts the degeneracy of the LLs (in the quantum number k_y) and broadens the formerly sharp LLs into bands, so-called (electric) Landau bands. The bandwidths for the Landau bands ($\sim 2|U_n|$) are therefore dependent on the Landau band index n, in an oscillatory manner, such that the electric-modulation-induced broadening of the energy spectrum is non-uniform.

For the case of a spatial magnetic modulation we consider the 2DEG to be subjected to the following magnetic field: $\boldsymbol{B} = (B + B_\mathrm{m}(x))\hat{e}_z$. Here B is the external uniform magnetic field applied along the z-direction while $B_\mathrm{m}(x)$ is the one-dimensional spatial magnetic modulation modulated along the x-direction. Again it will be modelled using a simple sinusoidal potential of the form $B_\mathrm{m}(x) = B_\mathrm{m}\cos(Kx)$ where B_m is the constant amplitude of the magnetic modulation and is assumed to be weak, i.e. $B_\mathrm{m} \ll B$. The single-electron Hamiltonian for the magnetically modulated system is given by $H^{\{\mathrm{m}\}} = (\boldsymbol{p} - e\boldsymbol{A})^2/(2m_b)$. Here $\boldsymbol{p}$ is the momentum operator and $\boldsymbol{A}$ the vector potential, while the letter 'm' is used to denote the magnetic case. Again choosing the Landau gauge $(\boldsymbol{A} = (0, xB + B_\mathrm{m}/K\sin(Kx), 0))$ this Hamiltonian can be written as $H^{\{\mathrm{m}\}} = H_0 + H_{B_\mathrm{m}}$. The second term is given by

$$H_{B_\mathrm{m}} = \frac{\omega_m}{K}(-\hbar k_y + eBx)\sin(Kx) + \frac{m_b\omega_m^2}{4K^2}(1 - \cos(2Kx)) \tag{2.3}$$

where we have written $\omega_m = eB_\mathrm{m}/m_b$ in analogy to the cyclotron frequency ω_c. Here the eigenvalues and eigenfunctions corresponding to this Hamiltonian cannot be solved analytically. Instead, since we are interested only in a weak magnetic modulation we take H_{B_m} to be a small perturbation of the 2DLS, H_0. Taking the $\psi_{n,k_y}(x, y)$ then as the unperturbed eigenstates, one calculates from first-order perturbation theory the eigenvalues for the weakly modulated magnetic system, $H^{\{\mathrm{m}\}}$, as [9]

$$E_{n,k_y}^{\{\mathrm{m}\}} = (n + 1/2)\hbar\omega_c + V_n \cos(Kx_0) \tag{2.4}$$

where only linear terms in B_m have been retained. Here

$$V_n = \hbar\omega_m/2[L_n^{(1)}(\chi) + L_{n-1}^{(1)}(\chi)]\exp(-\chi/2)$$

(see reference 16 of [28] for further discussion) while $L_n^{(\alpha)}(x)$ is an associated Laguerre polynomial.

In an analogous fashion to the case for the electrically modulated system, the application of a weak magnetic modulation similarly broadens the formerly sharp LLs into bands, so-called (magnetic) Landau bands. In this case the bandwidth of the Landau bands ($\sim 2|V_n|$) will once more have an oscillatory dependence on the Landau band index n such that the induced broadening of the energy spectrum is non-uniform.

An important feature of the newly acquired modulation-induced dispersion of these Landau bands is that they can become flat at particular magnetic field values. Flat bands for either modulation type requires that $\Lambda_n^{\{\varsigma\}} = 0$ where, for the sake of brevity, we have written

$$\Lambda_n^{\{\varsigma\}} = \begin{cases} U_n & \varsigma = \mathrm{e} \\ V_n & \varsigma = \mathrm{m}. \end{cases}$$

In the quasi-classical limit, where one considers small magnetic fields and has many Landau bands occupied, one is able to write down an approximate condition for flat bands [1, 2, 9]:

$$2R_c = a(\lambda_\varsigma \mp 1/4) \qquad \text{with } \lambda_\varsigma = 1, 2, \ldots. \tag{2.5}$$

Here the '$-$' case corresponds to $\varsigma = \mathrm{e}$ while the '$+$' case corresponds to $\varsigma = \mathrm{m}$. From this condition, the corresponding flat-band energies, $\varepsilon_{\lambda_\varsigma}$, may be estimated [8]:

$$\varepsilon_{\lambda_\varsigma} = \frac{1}{8}\left(\frac{a}{\ell}\right)^2\left(\lambda_\varsigma \mp \frac{1}{4}\right)^2\hbar\omega_c. \tag{2.6}$$

Similarly, broad bands occur when the dispersion of these Landau bands is a maximum, i.e. $\partial\Lambda_n^{\{\varsigma\}}/\partial\mathcal{X} = 0$. Again, in the quasi-classical limit one is able to write down an approximate condition for broad bands [1, 2, 9]:

$$2R_c = a(\lambda_\varsigma \pm 1/4) \qquad \text{with } \lambda_\varsigma = 1, 2, \ldots. \tag{2.7}$$

It is well known that in the absence of a modulation the DOS for the 2DLS consists of a series of equally spaced delta functions at energies equal to ε_n. The addition of a weak spatially periodic modulation however modifies the former delta function like the DOS by broadening the singularities at the energies ε_n into bands. In the limit where disorder broadening of the Landau bands is small compared to the modulation-induced broadening, the DOSs are given by [27, 28]

$$\mathcal{D}(\varepsilon) = \frac{A}{\pi\ell^2}\sum_{n,k_y}\delta[\varepsilon - \varepsilon_{n,k_y}] = \frac{A}{\pi^2\ell^2}\sum_{n=0}^{\infty}\frac{\theta(|\Lambda_n^{\{\varsigma\}}| - |\varepsilon - \varepsilon_n|)}{\sqrt{(\Lambda_n^{\{\varsigma\}})^2 - (\varepsilon - \varepsilon_n)^2}} \tag{2.8}$$

where $\theta(x)$ is a unit Heaviside step function. From equation (2.8) it can be seen that on either side of the low- and high-energy edges of the broadened Landau bands there exist one-dimensional van Hove singularities, which are of the inverse-square-root type, and are responsible for forming a double-peak structure in the DOS.

3. Equilibrium thermodynamic quantities

In the following we present detailed calculations for the electronic contribution to the equilibrium thermodynamical properties for a 2DEG in a perpendicular magnetic field which is subjected to additional spatially modulated periodic potentials in one dimension. The cases of both a weak electrically modulated potential and a weak magnetically modulated potential are treated. In particular, a detailed study of the chemical potential, internal energy, Helmholtz free energy, electronic entropy, electronic specific heat, orbital magnetization and orbital magnetic susceptibility are given. We employ the simplest model for a 2DEG which is achievable in semiconductor systems. Namely, we consider free non-interacting electrons, which lie in the (x, y) plane, within a single-electron approximation, and take the conduction band to be parabolic and spherically symmetric. To this a quantizing magnetic field is applied perpendicular to the plane of the 2DEG (i.e. along the z-direction) while an additional spatial potential is periodically modulated along the x-direction. Spin degeneracy of an electron is included but not the spin splitting since we are primarily interested in the behaviour of the system pertaining to small magnetic fields.

3.1. Chemical potential, internal energy, Helmholtz free energy

The magnetic-field- (B-) and temperature- (T-) dependent chemical potential $\mu \equiv \mu(B, T)$ of a system is determined through the normalization of the Fermi–Dirac distribution function, i.e.

$$f(\varepsilon) = \left[\exp\left(\frac{\varepsilon - \mu}{k_B T}\right) + 1\right]^{-1}$$

where k_B is Boltzmann's constant, by setting

$$N = \int_0^\infty \mathcal{D}(\varepsilon) f(\varepsilon)\,\mathrm{d}\varepsilon. \tag{3.1}$$

Here N gives the total number of electrons. From this equation it can be immediately seen that changes in the DOS for the modulated system will be reflected in changes in the form for the chemical potential over that of the unmodulated system. Upon substituting equation (2.8) into (3.1) we obtain

$$N = \frac{A}{\pi^2 \ell^2} \sum_{n=0}^{\infty} \int_{-1}^{1} \frac{\mathrm{d}x}{\sqrt{1 - x^2}} \left[\chi_n \exp(z_n^{\{\varsigma\}} x) + 1\right]^{-1}. \tag{3.2}$$

Here $\chi_n = \exp(\varepsilon_n - \mu)/(k_B T)$ while $z_n^{\{\varsigma\}} = |\Lambda_n^{\{\varsigma\}}|/(k_B T)$. Equation (3.2) represents a condensed form for writing the results as obtained from the two separate modulation types. Note that such a practice will be followed throughout for all the thermodynamic expressions which we are to present. For fixed electron concentration, the above integral equation gives $\mu(B, T)$ only implicitly and therefore, in general, must be solved for numerically. As a particular limiting case of equation (3.2), if we consider zero temperature and assume no overlap between the Landau bands (i.e. $2|\Lambda_n^{\{\varsigma\}}| < \hbar\omega_c$), then the integral can be evaluated in closed form. One can thus write explicitly for the chemical potential [28, 29]

$$\mu(B, 0) = (n_F + 1/2)\hbar\omega_c + |\Lambda_{n_F}^{\{\varsigma\}}| \sin\left[\pi\left\{\frac{\varepsilon_F^0}{\hbar\omega_c} - (n_F + 1/2)\right\}\right]. \tag{3.3}$$

Here n_F gives the band index at the highest occupied Landau band. The first term in equation (3.3) gives the magnetic-field-dependent chemical potential, at zero temperature, for the 2DLS in the absence of a modulation. In this case the chemical potential will be pinned to the last occupied LL except at integer filling factors $\nu = \pi\ell^2 n_s$. The second term of equation (3.3) gives an additional correction to the chemical potential which results from weakly modulated systems. One can see then that $\mu(B, 0)$ is confined within the highest occupied Landau band except again at integer filling factors.

The total internal energy, U, of the electrons is given by

$$U = \int_0^\infty \varepsilon\mathcal{D}(\varepsilon) f(\varepsilon)\, \mathrm{d}\varepsilon. \tag{3.4}$$

Once more, upon substitution of equation (2.8) into equation (3.4), explicit evaluation for the internal energy yields

$$U = \frac{A}{\pi^2\ell^2} \sum_{n=0}^{\infty} \int_{-1}^{1} \frac{\mathrm{d}x}{\sqrt{1-x^2}}\ (\varepsilon_n + x|\Lambda_n^{\{\varsigma\}}|)[\chi_n \exp(z_n^{\{\varsigma\}} x) + 1]^{-1}. \tag{3.5}$$

This integral, which is dependent on $\mu(B, T)$, again cannot in general be evaluated in closed form for arbitrary temperatures but instead must be evaluated numerically once $\mu(B, T)$ is known.

The Helmholtz free energy, F, directly tells one how to balance the conflicting demands of a given system between minimum internal energy and maximum entropy [30]. It is a very important computational function since it offers one of the easiest of methods for finding other thermodynamical properties of the system (as derivatives of F) once one has determined how to calculate F from the energy eigenvalues for the particular system. For non-interacting fermions, the Helmholtz free energy of the whole assembly of electrons in our system is given by [31]

$$F = \mu N - k_B T \int_0^\infty \mathcal{D}(\varepsilon) \ln\left[1 + \exp\left(\frac{\mu - \varepsilon}{k_B T}\right)\right]\, \mathrm{d}\varepsilon \tag{3.6}$$

which for our weak spatially periodically modulated systems, in one dimension, becomes

$$F = \mu N - k_B T \frac{A}{\pi^2\ell^2} \sum_{n=0}^{\infty} \int_{-1}^{1} \frac{\mathrm{d}x}{\sqrt{1-x^2}}\ \ln\left[1 + \chi_n^{-1} \exp(-z_n^{\{\varsigma\}} x)\right]. \tag{3.7}$$

From this equation for the Helmholtz free energy together with the expression determining the chemical potential, i.e. equation (3.2), one is able to determine all equilibrium thermodynamic properties for our weakly modulated systems as derivatives of F.

3.2. Entropy and specific heat

In the case for the electronic contribution to the entropy, S_{el}, it is most expeditiously obtained from the previous results calculated for the internal energy and the Helmholtz free energy of our system, via $S_{\mathrm{el}} = (U - F)/T$. The contribution to the electronic specific heat, C_{el}, can now be immediately obtained by differentiating S_{el} with respect to T, since $C_{\mathrm{el}} = T(\partial S_{\mathrm{el}}/\partial T)_{A,N} = -T(\partial^2 F/\partial T^2)_{A,N}$. Remembering that the chemical potential itself is also temperature dependent, it is therefore now just a somewhat tedious task to evaluate explicitly for C_{el}. After much algebraic manipulation we find

$$C_{\mathrm{el}} = k_B \frac{A}{\pi^2\ell^2} \left[\left\{L_2 - \frac{(L_1)^2}{L_0}\right\} + \frac{2}{k_B T}\left\{l_1 - \frac{L_1\ell_1}{L_0}\right\} + \frac{1}{(k_B T)^2}\left\{\text{Ł}_1 - \frac{(\ell_1)^2}{L_0}\right\}\right]. \tag{3.8}$$

Here

$$L_r = \sum_{n=0}^{\infty} \int_{-1}^{1} \frac{\mathrm{d}x}{\sqrt{1-x^2}} \left(\frac{\varepsilon_n - \mu}{k_B T}\right)^r \mathcal{B}_n^{(1)}(x)$$

$$\ell_r = \sum_{n=0}^{\infty} |\Lambda_n^{\{\varsigma\}}| \int_{-1}^{1} \frac{x \, \mathrm{d}x}{\sqrt{1-x^2}} \, \mathcal{B}_n^{(r)}(x)$$

$$l_r = \sum_{n=0}^{\infty} |\Lambda_n^{\{\varsigma\}}| \int_{-1}^{1} \frac{x \, \mathrm{d}x}{\sqrt{1-x^2}} \left(\frac{\varepsilon_n - \mu}{k_B T}\right)^r \mathcal{B}_n^{(r)}(x)$$

$$\mathit{L}_r = \sum_{n=0}^{\infty} |\Lambda_n^{\{\varsigma\}}|^2 \int_{-1}^{1} \frac{x^2 \, \mathrm{d}x}{\sqrt{1-x^2}} \, \mathcal{B}_n^{(r)}(x)$$

where $\mathcal{B}_n^{(r)}(x) = [\chi_n \exp(z_n^{\{\varsigma\}} x)]^r / [\chi_n \exp(z_n^{\{\varsigma\}} x) + 1]^{r+1}$ has been introduced.

The first double grouping of terms by the curly brackets in equation (3.8) are just those corresponding to the unmodulated case, in the limit of the modulation potential strength going to zero, and are modified in form due to the presence of the weak modulation potential. The second and third double groupings of terms by the curly brackets are new modulation-induced terms. It should be noted that the second grouping of terms are linearly dependent on the strength of the modulation amplitude while the third grouping of terms are quadratically dependent. Thus both groupings of modulation-induced terms will tend to zero in the limit of very weak modulation potential strengths.

3.3. Magnetization and magnetic susceptibility

The magnetic properties of a system are in the main due to the electrons present in the system. In the presence of an external magnetic field two effects are important for the magnetic properties of the system. Namely: (a) the electrons move in quantized orbits in the magnetic field and (b) the spins of the electrons tend to align parallel to the direction of the magnetic field. The orbital motion of the electrons gives rise to a contribution to the orbital magnetization and the orbital magnetic susceptibility, while the alignment of the electrons' spin with the external magnetic field gives rise to an additional spin magnetization and spin magnetic susceptibility. This additional spin part will not be considered here for reasons previously already mentioned.

The electronic contribution to the orbital magnetization, M, of the system is given by $M = -(\partial F/\partial B)_{A,N}$. Observing that the chemical potential itself is dependent on the magnetic field, then for the orbital magnetization of our weakly modulated system, by differentiation of equation (3.7) with respect to the magnetic field, one obtains

$$M = \frac{k_B T}{B} \frac{A}{\pi^2 \ell^2} \sum_{n=0}^{\infty} \int_{-1}^{1} \frac{\mathrm{d}x}{\sqrt{1-x^2}} \left\{ \ln\left[1 + \chi_n^{-1} \exp(-z_n^{\{\varsigma\}} x)\right] - \frac{\varepsilon_n}{k_B T} \left[\chi_n \exp(z_n^{\{\varsigma\}} x) + 1\right]^{-1} \right\} - k_B T \frac{A}{\pi^2 \ell^2} \sum_{n=0}^{\infty} \frac{\partial z_n^{\{\varsigma\}}}{\partial B} \int_{-1}^{1} \frac{x \, \mathrm{d}x}{\sqrt{1-x^2}} \left[\chi_n \exp(z_n^{\{\varsigma\}} x) + 1\right]^{-1}. \tag{3.9}$$

Here

$$\frac{\partial z_n^{\{\varsigma\}}}{\partial B} = \begin{cases} \pm \dfrac{1}{k_B T} \dfrac{\mathcal{X}}{2B} (U_n + 2U_n^{(1)}) & \varsigma = \mathrm{e} \\ \pm \dfrac{1}{k_B T} \left[\dfrac{V_n}{B_\mathrm{m}} + \dfrac{\mathcal{X}}{2B} (V_n + 2V_n^{(1)})\right] & \varsigma = \mathrm{m} \end{cases}$$

where

$$U_n^{(\beta)} = V_\mathrm{e} \exp(-\mathcal{X}/2) L_{n-\beta}^{(\beta)}(\mathcal{X})$$

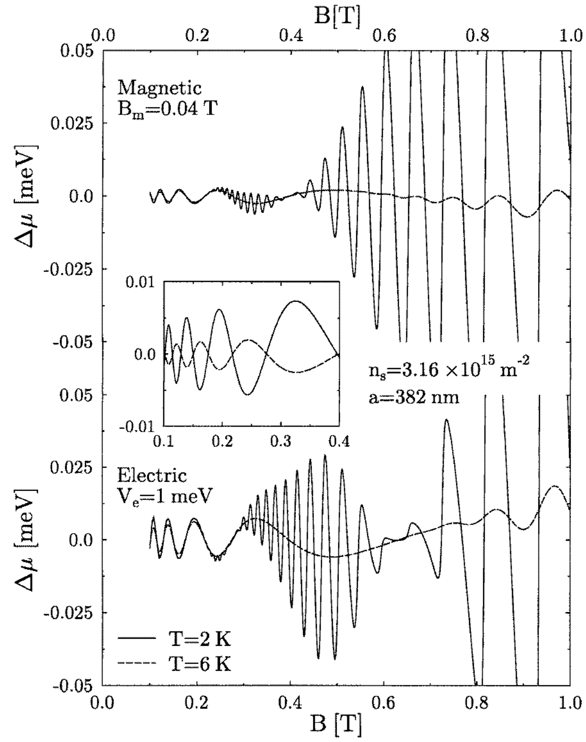

Figure 1. The change in the chemical potential versus the magnetic field, at finite temperatures ($T = 2$ K: full curve; $T = 6$ K: broken curve), due to an additional weak one-dimensional spatially periodic modulation potential which is either magnetic (top portion) or electric (bottom portion) in nature. Here $a = 382$ nm and $n_s = 3.16 \times 10^{15}$ m^{-2} are common to both modulated systems while $V_e = 1$ meV for the electrically and $B_m = 0.04$ T for the magnetically modulated systems. The inset shows the small-magnetic-field behaviour of $\Delta\mu$ (electric: full curve; magnetic: broken curve) versus B at $T = 6$ K only.

and

$$V_n^{(\beta)} = \hbar\omega_m \exp(-\mathcal{X}/2)[L_{n-\beta}^{(\beta+1)}(\mathcal{X}) - (1/2)L_{n-\beta}^{(\beta)}(\mathcal{X})].$$

The $\pm$ sign arises upon differentiation of a quantity where one was only previously interested in its modulus (i.e. $|\Lambda_n^{\{\varsigma\}}|$), whence the positive case is for $\Lambda_n^{\{\varsigma\}} > 0$ while the negative case is for $\Lambda_n^{\{\varsigma\}} < 0$. Again, the first two terms correspond to those found for the unmodulated case but modified in form due to the presence of a weak modulation. The third term represents the new modulation-induced term which disappears in the limit of the unmodulated system. Interestingly, it will be noted that the magnetically modulated case gives rise to an additional contribution to the orbital magnetization which is not present in the electrically modulated case. It is due to the magnetic modulation potential itself having a magnetic field dependence. The consequence of this additional term in the magnetically modulated case will be fully described in the following sections.

Finally, we are interested in the electronic contribution to the orbital magnetic susceptibility χ. It can be obtained directly from our previous results since $\chi = (\partial M/\partial B)_{A,N} = -(\partial^2 F/\partial B^2)_{A,N}$. Its explicit form will not be given here since it is a simple, but lengthy, extension of equation (3.9). It will be noted however that, as was the case for the orbital magnetization of the magnetically modulated system, the orbital magnetic susceptibility too also contains an additional contribution in the magnetically modulated case, as one now expects from performing a magnetic-field-dependent derivative.

4. Numerical results

A detailed numerical investigation of all of the thermodynamic properties, for both the electrically and magnetically modulated systems, is presented. All of the calculations that we have carried out are based on parameters typical for modulated 2DEG systems in GaAs. Specifically, we take $n_s = 3.16 \times 10^{15}$ m^{-2} and $a = 382$ nm. For the electrically modulated system we take $V_e = 1$ meV while for the magnetically modulated system we take $B_m = 0.04$ T. The effect of the modulation potential on any given quantity, say Π ($=\mu, U, F, \ldots$), will be given in terms of the difference between the modulated case and that for the unmodulated system, i.e. $\Delta\Pi = \Pi(V_e, B_m) - \Pi(V_e = B_m = 0)$.

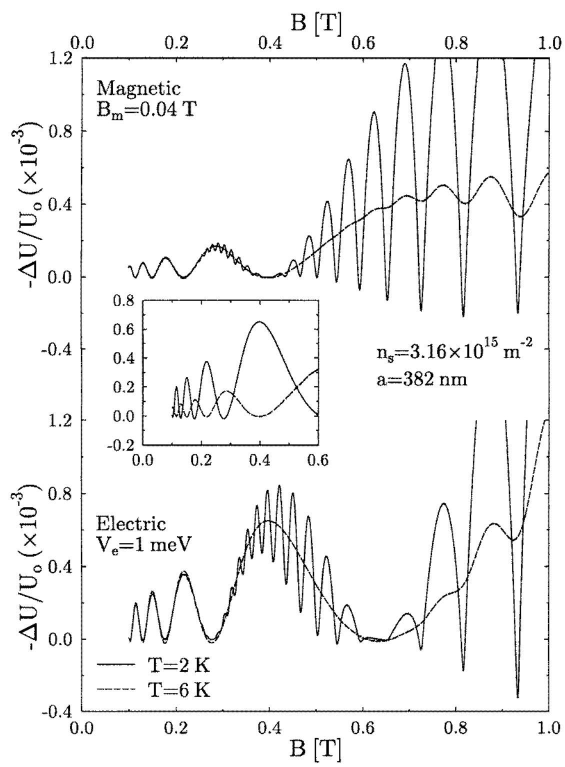

Figure 2. As figure 1, but now for the change in the internal energy versus the magnetic field. The y-axis has been scaled using $U_0 = N\varepsilon_F^0/2$ so that it is dimensionless.

In figures 1–7 we have plotted the changes in the various thermodynamic properties, $\Delta\Pi$, due to both a magnetic potential (top portion) and an electric potential (bottom portion) at the respective temperatures of $T = 2$ K (full curve) and $T = 6$ K (broken curve). In the inset in each of these figures we contrast the small-magnetic-field behaviours of the two modulation types (electric: full curve; magnetic: broken curve) at $T = 6$ K only. In figures 2–7, the $\Delta\Pi$ have been appropriately scaled so that they appear dimensionless.

In figure 1 we have plotted the change in the chemical potential, $\Delta\mu$, versus the

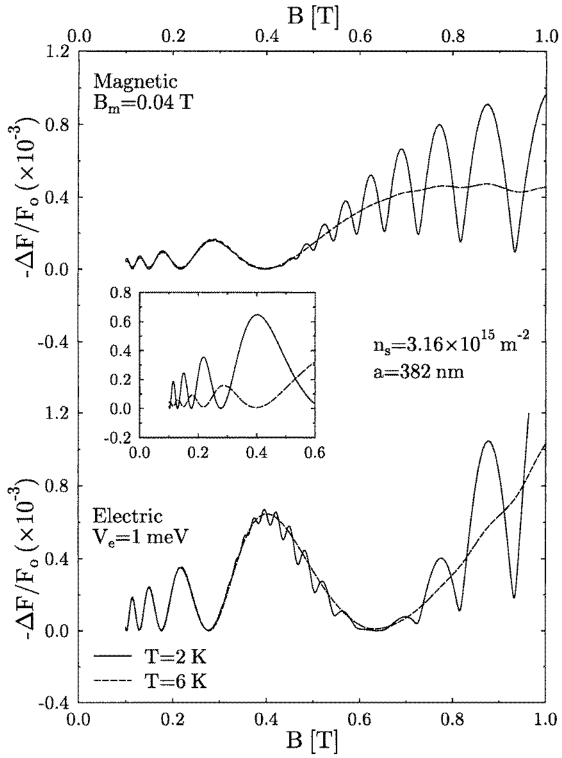

Figure 3. As figure 1, but now for the change in the Helmholtz free energy versus the magnetic field. The y-axis has been scaled using $F_0 = N\varepsilon_F^0/2$ so that it is dimensionless.

magnetic field. It is clearly seen that the effect of a weak one-dimensional periodic spatial modulation, is that it induces new oscillations, which appear for both modulation types, at low magnetic fields, and are only weakly temperature dependent. These modulation-induced oscillations have their origin in the commensurability of the two natural length scales present in such systems. They are similar to the oscillations initially observed in the magnetoresistance for such systems, at low fields, which have subsequently become known as 'Weiss' oscillations. Thus, any general commensurability oscillations in quantities other than the magnetoresistance will hereafter be referred to as Weiss-type oscillations. We see that zeros in $\Delta\mu$ occur for the flat-band conditions, as given by (2.5). Furthermore, since the chemical potential oscillates about the unmodulated result, between flat bands which correspond to one complete oscillation in the bandwidth, further zeros in $\Delta\mu$ due to this new Weiss-type oscillation occur. At higher temperatures such that the dHvA-type oscillations are no longer resolved (i.e. the $T = 6$ K curves) one sees that, surprisingly, the additional zeros occur about broad bands. At larger magnetic fields, both portions of figure 1 show dHvA-type oscillations. These are well known from the homogeneous 2DLS and are strongly damped with increasing temperature when compared to the low-field oscillations. The non-zero result at larger magnetic fields is also an indication that the effect of a weak one-dimensional modulation potential is not merely confined to the regime of low magnetic fields. The inset shows that the Weiss-type oscillations, at $T = 6$ K, for the two modulation types, are out of phase with one another by 180°. Note that the dHvA-type oscillations for the two modulation types remain in phase with each other.

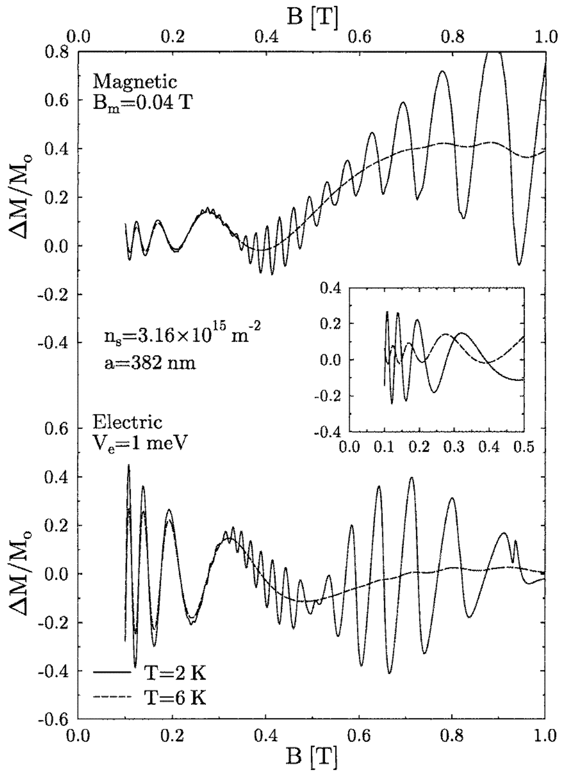

Figure 4. As figure 1, but now for the change in the orbital magnetization versus the magnetic field. The y-axis has been scaled using $M_0 = N\mu_B^*$ so that it is dimensionless.

In figures 2 and 3 we plot the change in internal energy, ΔU, and the change in the Helmholtz free energy, ΔF, respectively, versus the magnetic field. Both quantities have been scaled using $U_0 = F_0 = N\varepsilon_F^0/2$, so that the corresponding y-axes appear dimensionless. At small magnetic field strengths for either modulation type we clearly see, from both figures, that the effect of an additional modulation potential leads to new Weiss-type oscillations, with zeros occurring approximately for their respective flat-band conditions, and which are only weakly temperature dependent. Note that such oscillations between the two modulation types are 90° out of phase with one another. At higher magnetic fields the familiar dHvA-type oscillations occur and are strongly temperature dependent.

In figure 4 the change in the orbital magnetization, ΔM, versus the magnetic field is calculated according to equation (3.9). The change in the orbital magnetization has been scaled using $M_0 = N\mu_B^*$, where $\mu_B^* = e\hbar/(2m_b)$ is the effective Bohr magneton, in such a way that it once more appears dimensionless. Again, at low magnetic fields new Weiss-type oscillations are observed with zeros occurring for the respective flat-band conditions. Since the orbital magnetization also oscillates about the zero position, additional zeros in the orbital magnetization due to the Weiss-type oscillations result. These are most clearly seen from the $T = 6$ K curves when the dHvA-type oscillations have been all but washed away. For the electric case these additional zeros occur about magnetic field values given by the broad-band condition and are thus in phase with those for $\Delta\mu$, while for the magnetic case such zeros occur away from the magnetic field values given by its corresponding

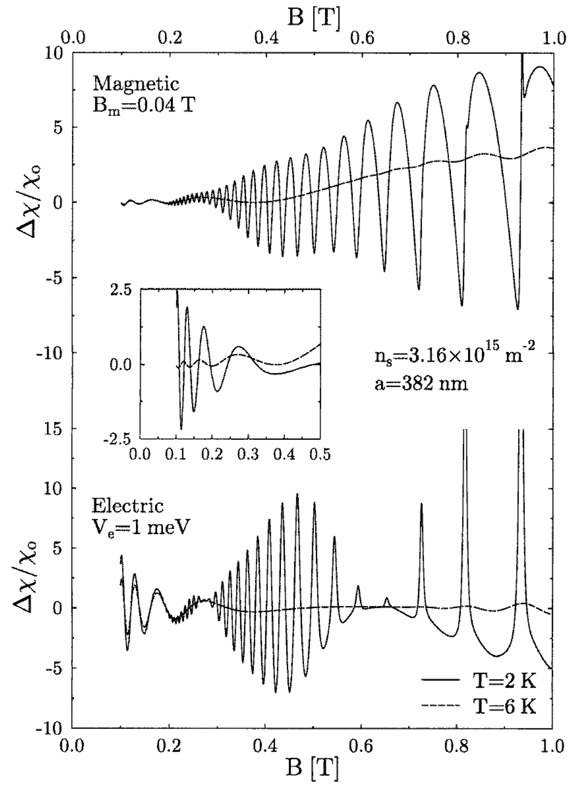

Figure 5. As figure 1, but now for the change in the orbital magnetic susceptibility versus the magnetic field. The y-axis has been scaled using $\chi_0 = N\mu_B^*/B$ so that it is dimensionless.

broad-band condition. This can be attributed to the additional magnetic-field-dependent term appearing in the orbital magnetization due to the magnetic field dependence of the modulation potential itself. As a result, there is no clearly defined phase difference in the Weiss-type oscillations between the two modulation types. This is clearly evident in the inset to figure 4. Again, these new Weiss-type oscillations are only weakly dependent on temperature and, at larger magnetic fields, dHvA-type oscillations for both modulation types are clearly seen. Similar results are obtained in figure 5 for the change in the orbital magnetic susceptibility, $\Delta\chi$, versus the magnetic field to those for the orbital magnetization. However, here the Weiss-type oscillations for the electric modulation have been shifted by 90° relative to the Weiss-type oscillations as found in both ΔM and $\Delta\mu$. For the magnetic modulation, as was the case with the orbital magnetization, no clearly discernible phase shift between the Weiss-type oscillations results due to the orbital magnetic susceptibility again containing additional magnetic-field-dependent terms since the modulation potential is itself dependent on the magnetic field. Both curves have been scaled by a factor of $\chi_0 = N\mu_B^*/B$ so that the y-axis appears dimensionless.

In figure 6 we have plotted the change in the electronic entropy, ΔS_{el}, versus the magnetic field. The scaling factor employed here was $S_0 = k_B N$. The appearance of Weiss-type oscillations at low magnetic fields is again predicted but these are shifted in phase by a factor of 90° relative to the Weiss-type oscillations appearing in $\Delta\mu$. They are again only weakly dependent on temperature. Once more a 180° phase difference exists

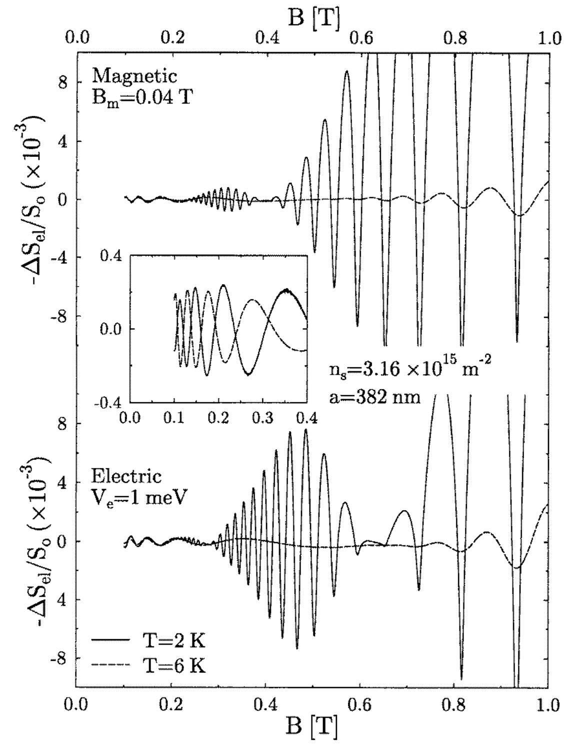

Figure 6. As figure 1, but now for the change in the electronic entropy versus the magnetic field. The y-axis has been scaled using $S_0 = Nk_B$ so that it is dimensionless.

between the two modulation types for such commensurability oscillations.

For the change in the electronic contribution to the specific heat, ΔC_{el}, versus the magnetic field, as obtained from equation (3.8) (see figure 7) similar results to those for ΔS_{el} are found. These oscillations are however 180° out of phase with the corresponding ones found in ΔS_{el} and 90° out of phase with those found in $\Delta\mu$. Again the curves have been scaled according to $(C_{el})_0 = k_B N$. In passing it will be noted that the additional Weiss-type oscillations appearing in both ΔS_{el} and ΔC_{el} at low magnetic fields are not large effects. Their respective amplitudes of oscillation are relatively small when compared to the amplitude of the dHvA-type oscillations, particularly at lower temperatures.

The electronic contribution to the specific heat, C_{el}, for the weakly modulated systems at large magnetic fields, differs significantly from that for the unmodulated 2DLS. In figure 8, C_{el} versus B is shown for a one-dimensional weakly modulated electric potential (full curve) and for the unmodulated system (broken curve) at $T = 2$ K. For the modulated system, at high magnetic fields an additional non-zero contribution to C_{el} results (seen as broadened peaks). Thus C_{el} for the modulated system is seen to consist of two contributions, whereas C_{el} for the unmodulated system consists only of a single contribution. At high magnetic fields, such that $\hbar\omega_c \gg k_B T$, the modulation-induced broadening of the LLs into bands lifts the k_y-degeneracy of the LL eigenspectrum and therefore allows for intra-Landau-band thermal excitations to contribute to C_{el}. Due to the k_y-degeneracy of the LL eigenspectrum in the unmodulated system, no such additional contribution to C_{el} can arise in this case. Such corresponding high-magnetic-field behaviour is also found in the weakly one-dimensional

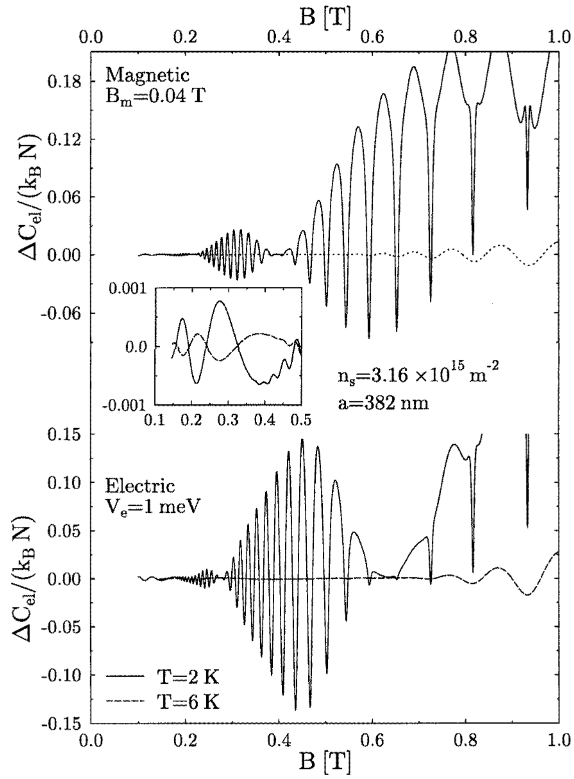

Figure 7. As figure 1, but now for the change in the electronic specific heat versus the magnetic field. The y-axis has been scaled using $(C_{\mathrm{el}})_0 = Nk_B$ so that it is dimensionless.

magnetically modulated system. At weaker magnetic fields, for both the modulated and the unmodulated systems, the inter-level thermal excitations begin to contribute to C_{el}, and they completely dominate in the limit of very low magnetic fields. These contributions are seen as the sharp spikes appearing in figure 8.

Interestingly, the situation for the modulated system is similar to that for an unmodulated system if one considers a phenomenological broadening of the LLs. The effect of Gaussian broadening of the LLs was considered by Zawadzki and Lassnig [24, 25]. Such a situation was found to exhibit similar intra-level contributions to our C_{el}, at higher magnetic fields.

Table 1. The phase differences of the Weiss-type oscillations as found in the thermodynamic quantities relative to those of the chemical potential.

	Phase shift	
Quantity	Electric	Magnetic
S	90°	90°
C_{el}	90°	90°
M	0°	—
χ	90°	—

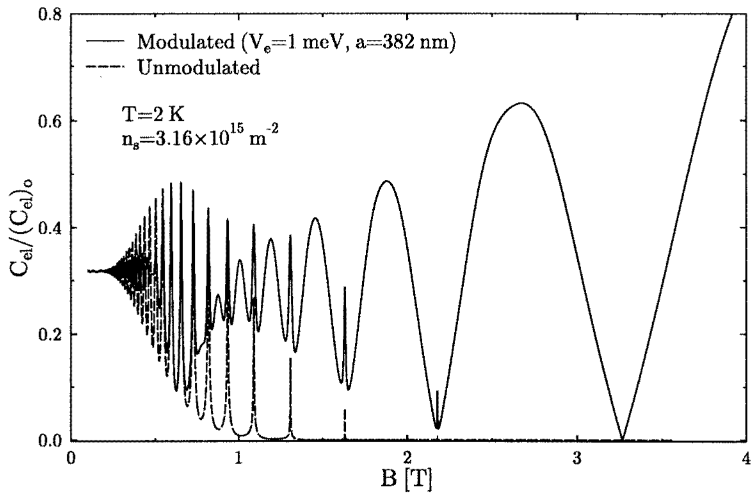

Figure 8. The normalized electronic specific heat versus the magnetic field at $T = 2$ K. The full curve is for the weakly modulated electric system ($V_e = 1$ meV, $a = 382$ nm) while the broken curve corresponds to the unmodulated two-dimensional Landau system. In both cases $n_s = 3.16 \times 10^{15}$ m^{-2}.

Table 2. The phase differences of the Weiss-type oscillations relative to those found between two specific quantities.

		Phase shift	
Quantity	Relative to	Electric	Magnetic
U	F	0°	0°
U	Bandwidth	0°	0°
F	Bandwidth	0°	0°
χ	M	90°	—
C_{el}	S	180°	180°

Table 3. The phase shifts in the Weiss-type oscillations occurring in the thermodynamic functions between the electric and magnetic modulations.

Quantity	Phase shift between the two modulation types
μ	180°
U	90°
F	90°
S_{el}	180°
C_{el}	180°
M	Not discernible
χ	Not discernible

To summarize, in table 1 we present the phase difference of the Weiss-type oscillations relative to the chemical potential μ while in table 2 the phase difference of the Weiss-type oscillations relative to another given quantity are presented. In table 3 the phase shifts in the Weiss-type oscillations occurring in the thermodynamic functions between the electric and magnetic modulations are given.

5. Asymptotic results

In this section we derive asymptotic expressions for the Helmholtz free energy, chemical potential and orbital magnetization, which are valid in the quasi-classical limit, and can account for the Weiss-type oscillations together with the more familiar dHvA-type oscillations which appear in these quantities.

To find the asymptotic expression for the Helmholtz free energy in this limit we begin with equation (3.6). This time however we shall employ an asymptotic result for the DOS which is valid in the limit of small magnetic fields when many Landau bands are occupied. In the case of a weak electric modulation, an approximate analytical formula for the DOS in the quasi-classical limit has been given by Zhang and Gerhardts [8]. Following their procedure verbatim for the case of a weak magnetic modulation, we find that we are able to write the DOS, to leading order in either modulation strength, as

$$\mathcal{D}(\varepsilon) \approx \frac{A}{\pi\ell^2}\frac{1}{\hbar\omega_c}\left\{1 - 2\cos\left(\frac{2\pi\varepsilon}{\hbar\omega_c}\right)\left[1 - \Omega^{\{\varsigma\}}(\varepsilon)\cos^2\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\varepsilon}{\hbar\omega_c}} - \frac{\pi}{4}\right)\right]\right\} \tag{5.1}$$

where

$$\Omega^{\{\varsigma\}}(\varepsilon) = \begin{cases} \dfrac{a}{\ell}\left(\dfrac{V_{\mathrm{e}}}{\hbar\omega_c}\right)^2\sqrt{\dfrac{\hbar\omega_c}{2\varepsilon}} & \varsigma = \mathrm{e} \\[2ex] \dfrac{1}{(2\pi)^2}\left(\dfrac{a}{\ell}\right)^3\left(\dfrac{\hbar\omega_m}{\hbar\omega_c}\right)^2\sqrt{\dfrac{2\varepsilon}{\hbar\omega_c}} & \varsigma = \mathrm{m}. \end{cases}$$

For definiteness we will only outline the calculation corresponding to the case for an electric modulation. For the case of a magnetic modulation, the calculation proceeds in an exactly analogous manner. Substitution of equation (5.1), for the asymptotic limit DOS, into equation (3.6) gives

$$\begin{aligned} F \approx \mu N &- \frac{A}{\pi\ell^2}\frac{k_BT}{\hbar\omega_c}\int_0^\infty \mathrm{d}\varepsilon\ \left[1 - 2\cos\left(\frac{2\pi\varepsilon}{\hbar\omega_c}\right)\right]\ln\left[1 + \exp\left(\frac{\mu-\varepsilon}{k_BT}\right)\right] \\ &- \frac{A}{\pi\ell^2}\frac{k_BT}{\hbar\omega_c}\frac{a}{\ell}\left(\frac{V_{\mathrm{e}}}{\hbar\omega_c}\right)^2\sqrt{2\hbar\omega_c}\int_0^\infty \frac{\mathrm{d}\varepsilon}{\sqrt{\varepsilon}}\ \cos\left(\frac{2\pi\varepsilon}{\hbar\omega_c}\right) \\ &\times\ \cos^2\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\varepsilon}{\hbar\omega_c}} - \frac{\pi}{4}\right)\ln\left[1 + \exp\left(\frac{\mu-\varepsilon}{k_BT}\right)\right]. \end{aligned} \tag{5.2}$$

The first of these integrals gives the contribution for the unmodulated 2DLS to the Helmholtz free energy in the quasi-classical limit. One will note that in this case, for zero broadening of the LLs, the approximate DOS at small magnetic fields is given by the well-known result $\mathcal{D}(\varepsilon) \approx m_b A/(\pi\hbar^2)\{1 - \cos[2\pi\varepsilon/(\hbar\omega_c)]\}$, and is clearly evident in this integral. The second of these integrals gives the correction to the Helmholtz free energy for the weakly modulated system. Both integrals, under the assumption that $2k_BT \ll \mu$, can be evaluated analytically at finite temperatures. The final result is (see appendix A)

$$F = \mu N + F_0 + F_{\mathrm{mod}}. \tag{5.3}$$

Here

$$F_0 \approx -\frac{A}{\pi\ell^2}\frac{(k_BT)^2}{\hbar\omega_c}\int_{-\mu/k_BT}^{\infty} \mathrm{d}u\ \ln(1+\mathrm{e}^{-u}) + \frac{A}{\pi\ell^2}\frac{\hbar\omega_c}{2\pi^2} - k_BT\frac{A}{\pi\ell^2}\frac{\cos[2\pi\mu/(\hbar\omega_c)]}{\sinh(T/T_c)} \tag{5.4}$$

and gives the asymptotic contribution to the Helmholtz free energy for the unmodulated 2DLS. It gives a pure oscillatory dHvA-type contribution to F_0 due to the magnetic-field-dependent cosine term. F_{mod} gives the asymptotic contribution to the Helmholtz free energy in the presence of the additional electric modulation potential. In terms of the difference between that of the modulated case and that of the unmodulated case, i.e. $\Delta F = F(V_\mathrm{e}) - F(V_\mathrm{e}=0) = F_{\mathrm{mod}}$, one has

$$\Delta F \approx -\frac{A}{\pi\ell^2}\frac{(k_BT_c)^2}{\hbar\omega_c}\frac{a}{\ell}\left(\frac{V_\mathrm{e}}{\hbar\omega_c}\right)^2\sqrt{\frac{\hbar\omega_c}{2\mu}}\pi^2 \times\left[1-\frac{T/T_c}{\sinh(T/T_c)}\cos\left(\frac{2\pi\mu}{\hbar\omega_c}\right)\right]\cos^2\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}}-\frac{\pi}{4}\right). \tag{5.5}$$

Here $T_c = \hbar\omega_c/(2\pi^2k_B)$ is known as the critical temperature. As is well known, it determines the amplitude of the dHvA-type oscillations at small magnetic fields (via the prefactor $(T/T_c)/\sinh(T/T_c)$) which, as expected, are exponentially damped as a function of the temperature. The critical temperature thus defines the temperature above which the dHvA-type oscillations will be washed away. Equation (5.5) can also account for the Weiss-type oscillations. At low magnetic fields, the cosine-squared term gives rise to new oscillations, that is Weiss-type oscillations, occurring in ΔF as an amplitude modulation of the dHvA-type oscillations such that zeros result when the electric flat-band condition is satisfied. However, no temperature dependence for these oscillations is obtained from our asymptotic expression. All information regarding the temperature dependence of the Weiss-type oscillations is lost since, by assuming $2k_BT \ll \mu$, one considers only the leading-order terms in $2k_BT/\mu$ (see appendix A). From our numerical results however, it is to be expected that by retaining terms of higher order in $2k_BT/\mu$ would produce only a weak temperature dependence in the Weiss-type oscillations.

For the magnetic case, a similar calculation to that given above leads to essentially the same result as equation (5.5); however, in this case the prefactor appearing outside the square brackets must be replaced. In general then, for both cases, one is able to write

$$\Delta F \approx -\Upsilon^{\{\varsigma\}}\left[1-\frac{T/T_c}{\sinh(T/T_c)}\cos\left(\frac{2\pi\mu}{\hbar\omega_c}\right)\right]\cos^2\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}}\pm\frac{\pi}{4}\right) \tag{5.6}$$

where

$$\Upsilon^{\{\varsigma\}} = \begin{cases} \dfrac{A}{\pi\ell^2}\dfrac{(k_BT_c)^2}{\hbar\omega_c}\dfrac{a}{\ell}\left(\dfrac{V_\mathrm{e}}{\hbar\omega_c}\right)^2\sqrt{\dfrac{\hbar\omega_c}{2\mu}}\pi^2 & \varsigma = \mathrm{e} \\ \dfrac{A}{\pi\ell^2}\dfrac{(k_BT_c)^2}{4\hbar\omega_c}\left(\dfrac{a}{\ell}\right)^3\left(\dfrac{\hbar\omega_m}{\hbar\omega_c}\right)^2\sqrt{\dfrac{2\mu}{\hbar\omega_c}} & \varsigma = \mathrm{m} \end{cases}$$

and where the '$-$' case corresponds to the electric case while the '$+$' case corresponds to the magnetic case. Clearly then in this limit, the phase shift in the Weiss-type oscillations between the two modulation types is 90°. The chemical potential in the asymptotic limit can now be readily found from equation (5.3) since $(\partial F/\partial\mu)_{N,A} = 0$.

The orbital magnetization in this asymptotic limit is found upon differentiating equation (5.3) (or its magnetically equivalent case) with respect to the magnetic field. The final result is $M = M_0 + M_{\rm mod}$. Here

$$M_0 \approx -\frac{1}{\pi^2}\frac{A}{\pi\ell^2}\frac{\hbar\omega_c}{B}\left[\exp\left(\frac{-\mu}{k_B T}\right)+1\right]^{-1} + \frac{A}{\pi\ell^2}\frac{k_B T}{B}\frac{\cos[2\pi\mu/(\hbar\omega_c)]}{\sinh(T/T_c)} + \frac{A}{\pi\ell^2}\frac{(k_B T)^2}{B}\frac{2\pi^2}{\hbar\omega_c}\frac{\cos[2\pi\mu/(\hbar\omega_c)]\cosh(T/T_c)}{\sinh^2(T/T_c)} + 2\pi\frac{A}{\pi\ell^2}\frac{\mu}{B}\frac{k_B T}{\hbar\omega_c}\frac{\sin[2\pi\mu/(\hbar\omega_c)]}{\sinh(T/T_c)} \tag{5.7}$$

is the asymptotic orbital magnetization for the 2DLS and gives only a pure dHvA-type contribution (from the magnetic-field-dependent sine and cosine terms) while

$$M_{\rm mod} \approx \frac{\Upsilon^{\{\varsigma\}}}{B}\left\{-\frac{2\pi\mu}{\hbar\omega_c}\frac{T/T_c}{\sinh(T/T_c)}\sin\left(\frac{2\pi\mu}{\hbar\omega_c}\right)\cos^2\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}}\pm\frac{\pi}{4}\right) + \left[1-\frac{(T/T_c)^2\cosh(T/T_c)}{\sinh^2(T/T_c)}\cos\left(\frac{2\pi\mu}{\hbar\omega_c}\right)\right]\cos^2\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}}\pm\frac{\pi}{4}\right) + \frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}}\left[1-\frac{T/T_c}{\sinh(T/T_c)}\cos\left(\frac{2\pi\mu}{\hbar\omega_c}\right)\right]\sin\left(\frac{4\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}}\pm\frac{\pi}{2}\right)\right\} + M_{\rm mod}^{\{\rm m\}} \tag{5.8}$$

gives the asymptotic contribution to the orbital magnetization in the presence of an additional modulation potential and represents an interference between the Weiss- and dHvA-type oscillations. For the electric case we choose the '$-$' case for which $M_{\rm mod}^{\{\rm m\}} = 0$ while for the magnetic case we choose the corresponding '$+$' case for which the additional magnetic-modulation-induced term is given by

$$M_{\rm mod}^{\{\rm m\}} = \frac{2\Upsilon^{\{\rm m\}}}{B_{\rm m}}\left[1-\frac{T/T_c}{\sinh(T/T_c)}\cos\left(\frac{2\pi\mu}{\hbar\omega_c}\right)\right]\cos^2\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}}+\frac{\pi}{4}\right). \tag{5.9}$$

From such an expression for the orbital magnetization, immediately it will be seen that one set of zeros in $\Delta M = M(V_{\rm e}, B_{\rm m}) - M(V_{\rm e} = B_{\rm m} = 0) = M_{\rm mod}$, due to the additional Weiss-type oscillations, occur for the respective flat-band conditions. What is not immediately clear however is where the addition zeros in ΔM occur, for either modulation type, due to such oscillations.

If in equation (5.6) one took the limit where the thermal broadening ($\sim k_B T$) is much larger than $\hbar\omega_c$ but less than the energetic spacing between adjacent flat bands, one would find

$$\Delta F \sim \Upsilon^{\{\varsigma\}}\cos^2\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}}\pm\frac{\pi}{4}\right). \tag{5.10}$$

This asymptotic expression for the change in the Helmholtz free energy describes the resulting Weiss-type oscillations which are independent of temperature. For this limit then, for the case of an electric modulation, by assuming that only the most rapidly varying factor in the magnetic field needs to be differentiated, we find for the change in the orbital magnetization

$$\Delta M^{\{\rm e\}} \sim \frac{2\pi^3}{B}\frac{A}{\pi\ell^2}\frac{(k_B T_c)^2}{\hbar\omega_c}\left(\frac{V_{\rm e}}{\hbar\omega_c}\right)^2\sin\left(\frac{4\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}}-\frac{\pi}{2}\right). \tag{5.11}$$

It is immediately obvious that the additional set of zeros occur at electric broad bands. For the magnetic modulation, when calculating the orbital magnetization for this case, one must consider the most rapidly varying factors in the magnetic field, both for the external magnetic field B and the modulation-dependent magnetic field B_{m}. When this is done we obtain

$$\Delta M^{\{\mathrm{m}\}} \sim \Upsilon^{\{\mathrm{m}\}} \cos\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}} + \frac{\pi}{4}\right) \times \left[\frac{2}{B_{\mathrm{m}}}\cos\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}} + \frac{\pi}{4}\right) + \frac{4\pi\ell}{aB}\sqrt{\frac{2\mu}{\hbar\omega_c}}\sin\left(\frac{2\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}} + \frac{\pi}{4}\right)\right]. \tag{5.12}$$

The additional set of zeros are determined by setting the square-bracket term equal to zero. When this is done the following transcendental equation results: $\theta = -\cot(\theta + \pi/4)B/B_{\mathrm{m}}$ where $\theta = 2\pi\ell/a\sqrt{2\mu/(\hbar\omega_c)}$. The phase shift occurring in the Weiss-type oscillations may be understood by plotting the following pair of equations; $y = \theta$ and $y = -\cot(\theta + \pi/4)B/B_{\mathrm{m}}$. The points of intersection between the two curves gives the additional roots. One finds that as the parameter θ (which is inversely proportional to the strength of the magnetic field) increases, the additional roots of $\Delta M^{\{\mathrm{m}\}}$ tend towards the asymptotes of the cotangent function which turn out to be those given by the magnetic broad-band condition. On the other hand, those roots which occur at small values for θ (corresponding to larger magnetic field values) are considerably removed from the asymptotes of the cotangent function and hence those values predicated from the magnetic broad-band condition. This accounts for the observed phase shift in the Weiss-type oscillations under an additional magnetic modulation compared to those oscillations found under an electric modulation, and is caused by the magnetic modulation potential itself giving an additional contribution to the orbital magnetization.

In the spirit of those approximations leading to equation (5.11), for the case of an electric modulation one is able to find a simple expression for the change in the orbital magnetic susceptibility:

$$\Delta\chi^{\{\mathrm{e}\}} \sim -\frac{8\pi^4}{B^2}\frac{A}{\pi\ell^2}\frac{\ell}{a}\frac{(k_B T_c)^2}{\hbar\omega_c}\left(\frac{V_{\mathrm{e}}}{\hbar\omega_c}\right)^2 \cos\left(\frac{4\pi\ell}{a}\sqrt{\frac{2\mu}{\hbar\omega_c}} - \frac{\pi}{2}\right). \tag{5.13}$$

The resulting Weiss-type oscillations (and hence its zeros) are therefore shifted in phase by a factor of 90° compared to those of the orbital magnetization.

From our asymptotic expression for the Helmholtz free energy we cannot find a similar asymptotic expression for the entropy which can account for the Weiss-type oscillations satisfactorily, since our asymptotic expression given by equation (5.3) gives no temperature dependence for such oscillations; a temperature derivative will therefore result in zero contribution to the Weiss-type oscillations.

Qualitatively it is to be expected that the Weiss-type oscillations can persist to higher temperatures and hence that the dHvA-type oscillations can no longer be resolved since, as pointed out by many authors [21, 27, 32, 33], at finite temperatures the thermal broadening ($\sim k_B T$) must remain smaller than the relevant energy scale responsible for each of the oscillation types. The dHvA-type oscillations occur due to the discreteness of the modulation-induced Landau bands. Such oscillations are therefore resolved provided that the thermal broadening remains less than the energy spacing between adjacent Landau bands, which is of the order of $\hbar\omega_c$. Weiss-type oscillations on the other hand are due to the size of the cyclotron diameter at the Fermi level versus the period of the modulation. Here then the relevant energy scale is the distance between adjacent flat bands. It can be estimated, from the flat-band energies $\varepsilon_{\lambda_\varsigma}$ (see (2.6)), as $\Delta\varepsilon_{\lambda_\varsigma}/(\hbar\omega_c) \sim ak_F/2$. For typical parameters $a \approx 350$ nm and $n_s \approx 3.1 \times 10^{15}$ m^{-2} one has $ak_F/2 \approx 25$. Such estimates (for either modulation type) are considerably larger than the Landau band separation energy of $\hbar\omega_c$ and this is why the Weiss-type oscillations survive to much higher temperatures compared with the dHvA-type oscillations.

6. Conclusion

We have presented a systematic investigation of the equilibrium thermodynamical properties, at finite temperatures, for a 2DLS under the influence of a weak spatially periodic potential which is modulated in one dimension. Expressions for both a weak electric and a weak magnetic modulation have been explicitly given for the case of a simple cosine potential. Such modulation potentials lead to a broadening of the two-dimensional Landau energy spectrum into bands whose widths oscillate as a function of the externally applied magnetic field. This oscillation in the Landau bandwidths reflects the commensurability of the natural length scales present in such systems, namely the magnetic length ℓ and the modulation period a. We have shown that for such modulated systems, new modulation-induced commensurability oscillations, at low magnetic fields, manifest themselves in all such thermodynamic quantities of the system. In particular, we have shown that such oscillations are to be found in the chemical potential, Helmholtz free energy, internal energy, electronic entropy, electronic specific heat, orbital magnetization and orbital magnetic susceptibility of such weakly modulated systems. They are similar in nature to the Weiss oscillations which are found in the magnetoresistance. Subsequently, we have shown that a whole class of Weiss-type oscillations are to be expected in the thermodynamic quantities of such weakly modulated systems. Such Weiss-type oscillations, which are only weakly temperature dependent, occur in the thermodynamic quantities at low magnetic fields as an amplitude modulation of the well-known dHvA-type oscillations, familiar from the homogeneous 2DLS which may or may not be resolved depending on the temperature. Characteristics pertaining to these new commensurability oscillations in the thermodynamic quantities have been summarized in tables 1 and 2. Interestingly, we find that well-defined phase relations exist between the two modulation types for the commensurability oscillations occurring in most of the thermodynamic functions except the orbital magnetization and the orbital magnetic susceptibility. These results are summarized in table 3. Finally, we have given asymptotic expressions for the chemical potential, Helmholtz free energy and orbital magnetization in the quasi-classical limit of small magnetic fields and small but finite temperatures, which show explicitly both the Weiss- and dHvA-type oscillations.

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Appendix A

Evaluation of the integrals appearing in equation (5.2) reduces to the common problem of the evaluation of an integral of the type

$$\mathrm{I}(\alpha, \beta; u_0) = \int_{-u_0}^{\infty} \mathrm{d}u \, \cos(\alpha u + \beta) \ln[1 + \mathrm{e}^{-2u}] \qquad u_0 \gg 1. \tag{A.1}$$

This integral can be performed analytically in the limit of large u_0. Integration by parts twice followed by replacement of the lower limit of integration by $-\infty$ (since $u_0 \gg 1$) leads to

$$\mathrm{I}(\alpha, \beta; u_0) \approx -\frac{2u_0}{\alpha} \sin(\beta - \alpha u_0) + \frac{2}{\alpha^2} \cos(\beta - \alpha u_0) - \frac{1}{\alpha^2} \int_{-\infty}^{\infty} \mathrm{d}u \, \frac{\cos(\alpha u + \beta)}{\cosh^2(u)} \tag{A.2}$$

which since [34]

$$\int_{-\infty}^{\infty} \mathrm{d}u \, \frac{\cos(\alpha u + \beta)}{\cosh^2(u)} = \frac{\pi \alpha \cos(\beta)}{\sinh(\pi \alpha)} \tag{A.3}$$

gives

$$\mathrm{I}(\alpha, \beta; u_0) \approx -\frac{2u_0}{\alpha} \sin(\beta - \alpha u_0) + \frac{2}{\alpha^2} \cos(\beta - \alpha u_0) - \frac{\pi \cos(\beta)}{\alpha \sinh(\pi \alpha)}. \tag{A.4}$$

The first integral, I_1, appearing in equation (5.2) may, upon the change of variable $2u = (\varepsilon - \mu)/(k_B T)$, be written as

$$I_1 = \int_{-\mu/(k_B T)}^{\infty} \mathrm{d}u \, \ln[1 + \mathrm{e}^{-u}] + 4k_B T \, \mathrm{I}\left(\frac{4\pi k_B T}{\hbar \omega_c}, \frac{2\pi \mu}{\hbar \omega_c}; \frac{\mu}{2k_B T} \right) \tag{A.5}$$

whence equation (5.4) immediately follows.

The evaluation of the second integral, I_2, appearing in equation (5.2) is somewhat more involved. By writing the two separate cosine terms as one, using basic trigonometric identities, employing the change of variable $2u = (\varepsilon - \mu)/(k_B T)$ and retaining only those terms to leading order in $2k_B T/\mu$, since $2k_B T \ll \mu$, one has

$$\begin{aligned} I_2 = {} & \frac{1}{2\sqrt{\mu}} \mathrm{I}\left(\frac{4\pi k_B T}{\hbar \omega_c}, \frac{2\pi \mu}{\hbar \omega_c}; \frac{\mu}{2k_B T} \right) \\ & + \frac{1}{4\sqrt{\mu}} \mathrm{I}\left(\frac{4\pi k_B T}{\hbar \omega_c}, \frac{2\pi \mu}{\hbar \omega_c} + \frac{4\pi \ell}{a} \sqrt{\frac{2\mu}{\hbar \omega_c}} - \frac{\pi}{2}; \frac{\mu}{2k_B T} \right) \\ & + \frac{1}{4\sqrt{\mu}} \mathrm{I}\left(\frac{4\pi k_B T}{\hbar \omega_c}, \frac{2\pi \mu}{\hbar \omega_c} - \frac{4\pi \ell}{a} \sqrt{\frac{2\mu}{\hbar \omega_c}} + \frac{\pi}{2}; \frac{\mu}{2k_B T} \right) \end{aligned} \tag{A.6}$$

from which, after careful algebraic manipulation, equation (5.5) follows.

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